From (2), (4), and (5) we have

$$V=\int_{-1}^{+1}\int_{0}^{r'}\int_{0}^{2\pi}\rho r'^2dr'dpd\omega'\left[\frac{1}{r}+P_1\frac{r'}{r^2}+P_2\frac{r'^2}{r^3}+\ldots+P_i\frac{r'^i}{r^{i+1}}+\right]$$

by making $\sin\theta'd\theta' = -dp$ (as it evidently is), and changing the sign of V. Since ω' is independent of r' and p, and the first term evidently the mass of the spheroid divided by r (equal to $m\div r$,)

$$V=\frac{m}{r}+2\pi\rho\int_{-1}^{+1}dp\left[\tfrac{1}{4}P_1\frac{r'^4}{r^2}+\tfrac{1}{5}P_2\frac{r'^5}{r^3}+\ldots+\frac{1}{i+3}P_i\frac{r'^{i+3}}{r^{i+1}}+\ldots\right]\ldots(29)$$

Let $r'=\dfrac{a\sqrt{(1-e^2)}}{\sqrt{[1-e^2(1-p^2)]}}$, and let $\dfrac{r'^{i+3}}{a^{i+3}}$ be developed in to a series of p–functions, so that

$$r'^{i+3}=a^{i+3}[F_0+F_1+F_2+\ldots+F_i+\ldots]\ldots.(30)$$

If this be substituted in (29) we see by (19) that every term, when integrated, except the one containing P_i will disappear. If we expand the value of r'^{i+3}, retaining only e^2, we shall find

$$r'^{i+3}=a^{i+3}\left[1-\frac{i+3}{2}e^2p^2\right]=a^{i+3}\left[\left(1-\frac{i+3}{6}e^2\right)-\frac{i+3}{2}e^3\left(p^2-\tfrac{1}{3}\right)\right]$$
$$=a^{i+3}[F_0+F_2].$$

From this we see that $i=0$ and $i=2$ are the only values to be used; and since there is no P_0 in (29), we have

$$V=\frac{m}{r}-\pi\rho\frac{a^5}{r^3}\int_{-1}^{+1}e^2P_2(p^2-\tfrac{1}{3})dp=\frac{m}{r}-\pi e^2\rho\frac{a^5}{r^3}\int_{-1}^{+1}\tfrac{3}{2}(p^2-\tfrac{1}{3})^2dp.$$

$$\therefore\qquad V=\frac{m}{r}-\frac{4}{15}\pi e^2\rho\frac{a^5}{r^3}=\frac{m}{r}-\frac{ma^2e^2}{5r^3}.\ \ldots\ldots\ldots(31)$$

The preceding discussion will help the student to understand the nature and uses of Laplace's *Coefficients* and *Functions* in their more general form as given in works on the figure of the earth and elsewhere. Some mathematical expressions contain curious properties.

RECENT MATHEMATICAL PUBLICATIONS.

COMMUNICATED BY G. W. HILL.

Gauss, C. F. Werke. Band VI. Herausgegeben von der koniglichen Gesellschaft der Wissenschaften zu Gottingen. Gottingen. 1874. 4to. 664 pp. 25 M.

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Konigsberger, Dr. Leo. Verlesungen uber die Theorie der elliptischen Functionen, nebst einer Einleitung in die allgemeine Functionenlehre. 2 Theilen. Leipzig. 1874. 8vo. 432, 219 pp.

INVENTION OF A NEW NUMERICAL SYSTEM.

BY FERDINAND EISSFELDT, BOSTON, MASS.

In our common decimal system, distinct characters are given to the numbers from one to ten; and it is very well known that instead of ten, any other number, for instance eight, or twelve or sixteen, or even two, may be selected for a base. Such systems have actually been calculated, but they have not come into use: the advantage not being sufficient to counterbalance the inconvenience of changing one system into another.

The subject may be treated, however, in quite a different manner yet. There is no necessity for taking any base at all, and the numbers may be made to progress in their own natural succession; or, to express it in other words, every number, even one, may be made to serve as a base for a certain time. This can be accomplished as follows.

Instead of dividing the original units by ten and ten, as in our common decimal system, we divide them in the natural succession of the numbers themselves, first one, then two, then three and so forth, and for each division we make a new mark in the second column. If in the first column a rest should remain, such rest can never be greater than the number of marks in the second column, because, if the rest was greater by one, a new mark would be made in the second column. The whole theory is based upon the fact that the rest in any column can never be greater than the number of marks in the next column. If the marks in the second third and all following columns, and if all the rests, are divided in the same way, that is, in the natural succession of the numbers 1, 2, 3; the numbers will appear in a very simple form, each consisting of a base aud a rest or correcting part, this rest or correcting part, however, is sometimes naught; thus:

One corrected by naught equals one, one corrected by itself equals two;

Two corrected by naught equals three, two corrected by one equals four, two corrected by itself equals five;

Three corrected by naught equals six, three corrected by one equals seven, three corrected by two equals eight, three corrected by itself equals nine;

Four corrected by naught equals ten, four corrected by one equals eleven, four corrected by two equals twelve, four corrected by three equals thirteen, four corrected by itself equals fourteen;